Radiation dominated implosion

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Inertical Confinement Fusion configuration model is analized for direct ignition without an ablator. The compression of the target pellet is neglected and rapid volume ignition is achieved by a laser pulse, which is as short as the penetraton time of the light across the pellet. The reflectivity of the target is assumed to be negligible, and the absorptivity is constant so that the light pulse can reach the opposite side of the pellet. The necessary pulse length and pulse strength is calculated.

I. INTRODUCTION

We consider a spherical pellet of Deuterium-Tritium (DT) fuel of initial outer radius of 1143 μm. At the National Ignition Facility (NIF) the pellet had a hole in the middle, to reach better compression, and it had a thin "ablator" layer, which reflected the incoming light [1–3]. In this experiment the target capsule was indirectly ignited by the thermal radiation coming from the gold Hohlaum. The Hohlraum was heated by the radiation of 192 lasers [4, 5]. In the direct drive OMEGA laser experiment the initial radius of the DT capsule was smaller, $R = 430$ μm [4]. The incoming and reflected light exercised a pressure and compressed the pellet at NIF, to about $R = 80$ μm just before ignition. At this moment the hole in the capsule is already filled in and the target density was compressed to about 300-700 g/cm^3 [6, 7]. Then this target showed the development of Rayleigh-Taylor instabilities, which reduced the efficiency of ignition.

The initial compression pulse ("low foot") had lower frequency of or longer wavelength of 100-300 nm, which therefore had a higher reflectivity on the target, and led to compression. The reflectivity of light is high (> 0.6) for lower frequency light and decreasing with increasing frequencies. It becomes negligible at $\hbar\omega = 1$ keV. The compression pulse was followed by a shorter, higher frequency ignition pulse. The higher frequency pulse has negligible reflectivity and decreasing absorptivity, having $\alpha_K = 10^6$ cm^{-1} at $\hbar\omega = 20$ eV and $\alpha_K = 10$ cm^{-1} at $\hbar\omega = 1$ keV. If we take a higher frequency, shorter wavelength of 20 nm in the X-ray range then the absorptivity of the DT fuel is about $\alpha_K = 10^4$ cm^{-1} [8]. This means that the full pulse energy is absorbed in 10^{-4} cm $= 1\mu$m. That is in a thin surface layer. The internal domain is heated up due to adiabatic compression, up to ignition, but the major part of the approximately 10 μm thin compressed surface layer remains cold and only a 1 μm is heated up at the outside surface. See Fig. 8 of Ref. [8].

II. CONSIDERATIONS FOR THE TARGET

In our following consideration we could take (a) a compressed smaller initial state of radius $R = 80$ μm, which is then not compressed further but heated up further with a short penetrating light pulse. Alternatively we could (b) consider a solid, not compressed ball of the same amount of DT fuel, which is then made transparent and ignited by a laser pulse without compression of radius $R = 640$ μm,. In this second case due to the smaller density we will need a more energetic short pulse but also 8 times longer.

Reference [9] used a similar size target, with an outer ablator layer and initial compression. However, they used a special cone-in-shell configuration of the target. Through doping the target with Cu they were able to image the K-shell radiation of the target when it was radiated by an ultraviolet driver beam. From their images in Figure 2 d-f of [9], we see that ignition is achieved in an area of approximately 50 μm radius from the center of the target. By using a high-contrast laser and a 40 μm cone tip they were able to increase the fast electron coupling to the core from < 5% to 10-15% by an increase in the core-density and decrease in the source-to-core distance. If this laser-to-electron conversion efficiency would be further increased, the total laser energy coupled to the core would also increase above 15%, which may be good if we want to achieve fast-ignition inertial-confinement-fusion.

Here we can consider another configuration, without ablator layer and without pre-compression, using the early examples in Refs. [10, 11]. Then we have a more dilute target fuel with about 640 μm radius, case (b). With a deuterium-tritium ice as fuel, the target density is 1.062 g/cm^3. This target a priory has smaller absorptivity. If we want to absorb the whole energy of the incoming laser light on ~ 1.3 mm length, we need an absorptivity of $\alpha_K \approx 8$ cm^{-1}. This is about the absorptivity of DT fuel for soft X-ray radiation of 1 nm wavelength. Longer wavelength radiation would have a larger absorptivity, and would be absorbed in the outside layers of the pellet.

III.   SIMPLIFIED MODEL AND ITS EVALUATION

Consider a spherical piece of matter (E), which is sufficiently transparent for radiation. The absorptivity of the target matter is considered to be constant, such that the total energy of the incoming light is observed fully when the light reaches the opposite edge of the spherical target. This matter undergoes an exothermic reaction if its temperature exceeds T_c.
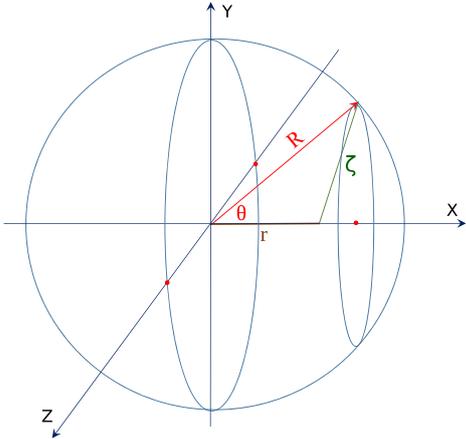

FIG. 1.   (color online) The sphere of the fuel, with an internal point at radius r. Let us chose the x-axis so that it passes through the point at r and the center of the sphere. Then let us chose a point on the sphere, and the angle of this point from the x-axis is denoted by Θ. Then the length between this surface point and the internal point at r is $\zeta = (R^2 + r^2 - 2Rr\cos\Theta)^{1/2}$. The propagation time from the surface point to the point at r equals $\tau = \zeta/c$.

The target matter is surrounded by an set of spherically distributed lasers, which emit the radiation necessary to heat up E. We are neglecting the expansion of the outer shell inwards as well as the expansion of the core, so that the core radius R is constant. We will measure the length in units of μm, and the time in units of μm/c.

We intend to calculate the temperature, T, distribution within the sphere, as a function of time, t, and the radial distance from the center of the sphere, i.e. radius r. We have two steps of the evaluation:

(i) In the 1st step we calculate that from the outside surface of the sphere how, much energy can reach a given point at r. Here we have to take into account that the outside thermal radiation starts at time, $t = 0$, so there is no radiation before, and only those parts of the outside surface can reach a point inside the sphere at time t, and which are on the backward light-cone of the point at r and time t. The integral for the energy density reaching the point from this part of the two dimensional outside surface of the sphere in unit time interval, dt, is $dU(r,t)/dt$.

(ii) Then we have to add up the accumulated radiation at position r, for the previously obtained energy and to obtain the time dependence of the temperature distribution, $T(r,t)$, we have to integrate $dU(r,t)/dt$ from $t = 0$, for each spatial position.

We perform the surface integral of step (i) in terms of integration for the proper time of the radiation with a delta function, selecting the surface element, which can reach the given internal point at a time.

Let us study a point within the sphere, at a distance r from the center. Choose the x-axis passing through this point and the center of the sphere. See Fig. 1.

The surface area of a ring of the sphere at the selected polar angle Θ is $dS = 2\pi R^2 \sin\Theta\, d\theta$.

Step (i):
At a point at r we receive radiation from a layer edge ribbon at time τ. The radiation at distance ζ is decreasing as $1/\zeta^2$. The total radiation reaching point r from the ribbon at Θ is

$$dU(r,t) \propto \frac{1}{\zeta^2}\delta(\zeta - \sqrt{R^2 + r^2 - 2rR\cos\Theta})\ , \qquad (1)$$

where $\tau = \zeta/c$, and we should integrate this for the surface of all ribbons.

The average intensity of thermal radiation reaching the surface of the pellet amounts to Q per unit surface (μm^2) and unit time (μm/c). Let us take a typical value for the energy of the total ignition pulse to be 2 MJ, in time 10 ps, then $Q = 2\text{MJ}\,(4\pi\cdot 640\mu\text{m})^{-2}\,(10\text{ps})^{-1}$ or $Q = 3.08\cdot 10^{19}\ \text{W/cm}^2 = 1.03\cdot 10^9\ \text{J\,c/cm}^3$.

Up to a given time t, the light can reach a space-time point (r,t), inside the sphere from different points of the outside surface, which were emitted in different times. At early times it may be that none of the surface points are within the backward light-cone of the point (r,t). At later times, from part of the surface points the light can reach (r,t), while at times larger than $2R/c$ all internal points can be reached from any surface point of the sphere. Thus, we calculate first what energy density, $U(r,t)$, we get at a space-time point (r,t), from earlier times. At a given point at r measured from the center of the sphere (assuming that a constant fraction, α_K, of the radiation energy is absorbed in unit length): [1]

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[1]   We are using the relation $\delta[g(x)] = \sum_i [(1/|g'(a_i)|)\delta(x - a_i)$ where a_i-s are the roots of $g(x) = \zeta - \sqrt{R^2 + r^2 - 2rRx}$, i.e. $g(a_1) = 0$. Now $a_1 = (R^2 + r^2 - \zeta^2)/(2rR)$ and $g'(x) = rR/\zeta$ so that the integrand is $\zeta/(\zeta^2 rR) = 1/(rR\zeta)$. The variable ζ depends on x (or Θ), so we should set the integral boundaries in terms of ζ accordingly.

$$dU(r,t) =$$
$$\alpha_K Q \int_0^t d\tau \, 2\pi R^3 \int_0^\pi d\cos\Theta \, \frac{\delta(\zeta - \sqrt{R^2 + r^2 - 2rR\cos\Theta})}{R^2 + r^2 - 2rR\cos\Theta}$$
$$= \alpha_K Q \int_0^t d\tau \, 2\pi R^3 \int_1^{-1} dx \, \frac{\delta(\zeta - \sqrt{R^2 + r^2 - 2rRx})}{R^2 + r^2 - 2rRx}$$
$$= 2\pi R^3 \alpha_K Q \cdot (Rr)^{-1} \int_{(R-r)/c}^{aR/c} \frac{d\tau}{\tau c} \, , \qquad (2)$$

where the integral over dx gives $1/(Rr\zeta) = (R\,r\,\tau\,c)^{-1}$. The time, $d\tau$, integral runs from the nearest point of the backward light cone to the surface of the sphere to the furthest point, aR/c. Here the parameter a will be described later. See Fig. 2.

Now we introduce a new, dimensionless time variable:

$$q \equiv \tau c / R$$

. Thus,

$$dU(r,t) = 2\pi R^3 \alpha_K Q \cdot (rRc)^{-1} \int_{1-r/R}^a \frac{dq}{q}$$
$$= 2\pi R^2 \alpha_K Q \cdot (rc)^{-1} \left[\ln(q)\right]_{1-r/R}^a \qquad (3)$$

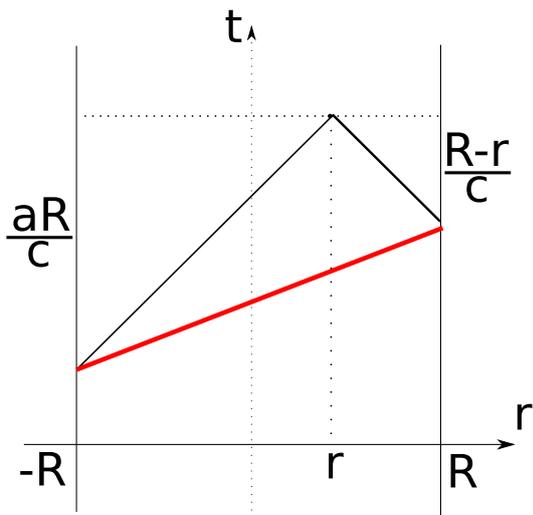where $\zeta = \tau c$ and a is the upper boundary of the integral

FIG. 2. (color online) The boundaries of the integration domains depending on r and t. The domain for the smallest τ-values cannot receive radiation, Eq. (2c), because the radiation started at $(R-r)/c$ earlier and it reaches the internal point at r later, At the same time the radiation from the opposite side reaches the point r also in time aR/c. The contour of the intersection of the backward light-cone with the surface of the sphere is indicated with the thick, red line. When the time from the momentum of ignition is longer than $t = 2R/c$ the radiation reaches the matter from all sides at every location r. At earlier times the upper boundary of integration should be evaluated. See Eq. (4).

over the dimensionless time dq

$$a = \begin{cases} 1 - r/R \, , & q < 1 - r/R \\ q \, , & 1 - r/R < q < 1 + r/R \\ 1 + r/R \, , & q > 1 + r/R \end{cases} \qquad (4)$$

Here actually the integral over dq is adding up the contributions of those surface elements of the sphere, from where radiation reaches the internal point at r at the same dimensionless time q. In the first case the radiation does not reach the point at r then, in the second part the radiation from the closest point of the sphere reaches r but from the opposite point not yet, in the third case radiation reaches r from all sides.

Thus the energy deposited in unit time at dimensionless time q is

$$dU(r,q) = \frac{2\pi R^2 \alpha_K Q}{rc} \times$$
$$\begin{cases} \ln[(1 + r/R)/(1 - r/R)], & q > 1 + r/R \\ \ln[q/(1 - r/R)], & 1 - r/R < q < 1 + r/R \\ 0, & q < 1 - r/R \end{cases}$$
$$\qquad (5)$$

Step (ii):
Neglecting the compression and assuming constant specific heat c_v, we get that $k_B \, dT = \frac{1}{n c_v} dU \cdot dq$, where k_B is the Boltzmann constant, and so

$$k_B T(r,t) = \frac{1}{n c_v} \int_0^{tc/R} dq \cdot dU(r,q) = \frac{2\pi R^2 \alpha_K Q}{n c_v \, rc} \times$$
$$\begin{cases} [q \ln(1 + r/R)/(1 - r/R))]_{1+r/R}^{tc/R} + \\ \quad (1 + r/R)\ln[(1 + r/R)/(1 - r/R)] - 2r/R \, , \\ \qquad \text{if:} \quad tc/R > 1 + r/R \\ [q \ln(q/(1 - r/R)) - q]_{1-r/R}^{tc/R} \, , \\ \qquad \text{if:} \quad 1 - r/R < tc/R < 1 + r/R \\ 0 \, , \qquad \text{if:} \quad tc/R < 1 - r/R \end{cases}$$
$$= H \cdot \frac{R^2}{r} \times$$
$$\begin{cases} tc/R \ln[(1 + r/R)/(1 - r/R)] - 2r/R \, , \\ \qquad \text{if:} \quad tc/R > 1 + r/R \\ tc/R \ln[tc/R/(1 - r/R)] - tc/R + 1 - r/R \, , \\ \qquad \text{if:} \quad 1 - r/R < tc/R < 1 + r/R \\ 0 \, , \qquad \text{if:} \quad tc/R < 1 - r/R, \end{cases}$$
$$\qquad (6)$$

where the number density of uncompressed DT ice is $n = 3.045 \cdot 10^{22}$ cm^{-3}, and the leading constant, H, is

$$H \equiv \frac{2\pi Q}{c \, c_V} \frac{\alpha_K}{n} = 6.8 \cdot 10^{-13} \text{J/cm.} \qquad (7)$$

Thus it follows:

$$k_B T(r,t) \propto \begin{cases} 0 \, , \qquad \text{if:} \quad tc/R < 1 - r/R \\ \frac{tc}{r}\left(\ln \frac{tc/R}{1 - r/R} - 1\right) + \frac{1 - r/R}{r/R} \, , \\ \qquad \text{if:} \quad 1 - r/R < tc/R < 1 + r/R \\ \frac{tc}{r} \ln \frac{1 + r/R}{1 - r/R} - 2 \, , \quad \text{if:} \quad tc/R > 1 + r/R \end{cases}$$
$$\qquad (8)$$

The surface of the discontinuity is characterized by the $T(r,t) = T_c$ contour line. If T_c is the ignition temperature, then here the DT ignition takes place on this contour line in the space-time. The tangent of this line is if $tc > R + r$:

$$\left(\frac{\partial r}{c\partial t}\right)_{T_c} = \left(\frac{\partial T}{c\partial t}\right)_{T_c} \bigg/ \left(\frac{\partial T}{\partial r}\right)_{T_c}$$

$$= \ln\frac{1 + r/R}{1 - r/R} \bigg/ \left\{\left[\frac{2tc}{R - r} - \frac{tc}{r}\ln\frac{1 + r/R}{1 - r/R}\right]\right\} \quad (9)$$

So the point (r_c, t_c) where the spacelike and timelike parts of the surface meet:

$$\left(\frac{\partial r}{c\partial t}\right)_{T_c} = 1 \rightsquigarrow t_c = \left\{\frac{2c}{R - r_c}\left[\ln\frac{1 + r_c/R}{1 - r_c/R}\right]^{-1} - \frac{c}{r_c}\right\}^{-1}$$

$$(10)$$

This line $t = t_c(r_c)$ separates the Space-like and Time-like branch of the discontinuity of $T(r,t) = T_c$. The
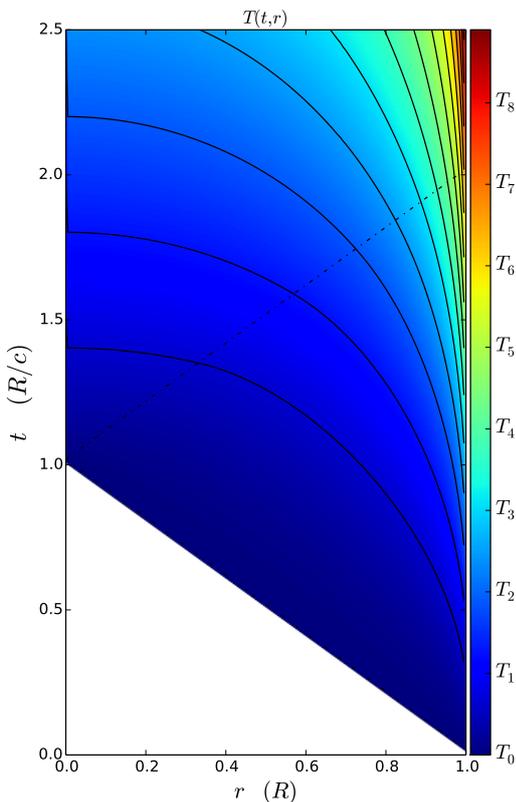

FIG. 3. (color online) The temperature distribution in function of distance and time. The dotted lines represent the light cone. The temperature is measured in units of $T_1 = H \cdot R = 272\,\text{keV}$, and $T_n = n \cdot T_1$.

discontinuity initiates at $r = R$ and $t = 0$ and it propagates first slowly inwards. Due to the radiative heat transfer the contour line of ignition, $T(r,t) = T_c$, accelerates inwards and at $r_c = r_c(T_c)$ it develops smoothly from space-like into a time-like discontinuity.

The same type of gradual development from space-like into time-like detonation may occur in the last phase of ultra-relativistic heavy ion collisions. If we include radiative heat transfer in a scenario described in our ref., the transition from space-like to time-like deflagration will be gradual. This, however, requires more involved numerical calculations.

IV. CONCLUSIONS AND DISCUSSIONS

In this model estimate, we have neglected the compression of the target solid fuel ball, as well as the reflectivity of the target matter. The relatively small absorptivity made it possible that the radiation could penetrate the whole target. With the model parameters we used the characteristic temperature was $T_1 = 272\,\text{keV}$, which is largr than the usually assumed ignition temperature, while our target is not compressed so the higher temperatures may be necessary to reach ignition according to the Lawson criterion. If we can achieve ignition at somewhat lower temperature than T_1, the ignition surface in the space time includes a substantial time-like hyper-surface, where instabilities cannot develop, because neighboring points are not causally connected.

From looking at the temperature lines in function of distance and time (figure 3), we see that the detonation at a higher critical temperatures, $T_c > T_3$ occurs after the radiation reaches from the other side. In this case, in the outside domain of the target, the inward propagation of the ignition front is slower than the speed of light (the dotted line on the figure indicates the speed of light reaching from the other side of the pellet). This makes it possible the development of instabilities in this region.

The most optimal configuration is if we achieve ignition at $T_c = T_1$ or slightly below. This leads to the fastest complete ignition of the target with the least possibility that instabilities occur.

An alternative possibility to apply this model is to consider a pre-compressed, more dense target, which is transparent and larger absorptivity. In this situation the ignition temperature is somewhat smaller, but we still can optimize the pulse strength and pulse length to achieve the fastest complete ignition of the target.

We can see if we neglect the importance of the speed of light, the theory would be far-fetched from reality. It is important to use the proper relativistic treatment to optimize the fastest completer ignition, with the least possibility of instabilities, which reduce the efficiency of ignition.

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